MAGNETIC FIELD OF A BRANCHING DISCHARGE

V.V. Borisov$^1$ and A.B. Utkin$^2$

$^1$ Research Institute of Physics, St. Petersburg University, St. Petersburg, Russia
$^2$ Institute for Laser Physics, Sci. Center “S.I. Vavilov State Optic Institute”, St. Petersburg, Russia

ABSTRACT: We consider space-time representation to the magnetic field produced by a right-angle branching discharge. Current pulse starts and propagates to the T-branching point. Then one discharge segment carries a part of the initial current in the original direction to the first (cloud) terminal point. The rest of the current pulse propagates in the vertical direction to the other (ground) terminal point. This scheme can serve as a rough model of complicated cloud-to-cloud discharge that partially branches to the ground. The report was stimulated by observation of corresponding natural phenomenon in July 1996 at the Ladoga Lake.

The discharge structure is shown in Figure 1. The current pulse starts at the point $O_l$ and propagates to the T-junction point $O_H$ at which it branches. One segment carries the part $\kappa$ of the initial current to the terminal point $F$. The rest of the current $(1-\kappa)$ is directed at a right angle to the terminal point $O$. According to the superposition principle, the total field produced by a discharge of this shape is a sum of three partial fields of radiating segments $O_lO_H$, $O_HF$, and $O_HO_l$. To find these fields we consider some segment $z \in [0, l]$ carrying current density

$$j = j_0 z, \quad j = \frac{1}{2\pi} \frac{\delta(\rho)}{\rho} (h(\rho) h(l-z) h(z-\beta(z-T)) l(z, \tau))$$

(1)

Figure 1. Structure of the discharge and field components.
which is a pulse of duration $T$ starting at the point $z=0$ at the moment $\tau=0$ ($\tau = ct$ is the time variable measured in the units of length, $c$ is the velocity of light) and propagating along the $z$ axes to the terminal point $z=l$ with the velocity $c \beta$. The term $I(z, \tau)$ is the continuous part of the current. We use the cylindric coordinate system $\rho, \phi, z$.

As it is shown in [Borisov and Utkin, 1995], the magnetic induction $B = B_\phi(z, \rho, \tau, l) e_\phi$ can be found with the help of the Bromwich-Borgnis potential $\Phi$ as

$$B_\phi(z, \rho, \tau, l) = 0 \quad \text{for} \quad \tau < r \quad \text{and} \quad \tau > T + \frac{1}{\beta} + \eta,$$

$$B_\phi(z, \rho, \tau, l) = -\frac{\partial}{\partial \rho} \Phi(z, \rho, \tau, l) \quad \text{for} \quad r < \tau < T + \frac{1}{\beta} + \eta,$$

$$\frac{\partial \Phi}{\partial \tau}(z, \rho, \tau, l) = \frac{1}{c} \int_{\Phi_3(z, \rho, \tau, l)} d\tau' \frac{1}{r^2 + (z - \tau')^2} \left( \begin{array}{c}
(z', \tau - \sqrt{r^2 + (z - \tau')^2})
\end{array} \right).$$

The integral limits for different interrelations between $z, \rho$, and $\tau$ are given in the table below:

<table>
<thead>
<tr>
<th>Condition</th>
<th>$\Phi_1$</th>
<th>$\Phi_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r &lt; \tau &lt; \min\left(T + r, \frac{1}{\beta} + \eta\right)$</td>
<td>0</td>
<td>$\frac{\beta}{\sqrt{1 - \beta^2}} (\tau - r) - \frac{\beta}{\sqrt{1 - \beta^2}} (\tau - r)$</td>
</tr>
<tr>
<td>$T + r &lt; \tau &lt; \frac{1}{\beta} + \eta$</td>
<td>$\frac{\beta}{\sqrt{1 - \beta^2}} (\tau - r)$</td>
<td>0</td>
</tr>
<tr>
<td>$\frac{1}{\beta} + \eta &lt; \tau &lt; T + r$</td>
<td>$\frac{\beta}{\sqrt{1 - \beta^2}} (\tau - r)$</td>
<td>0</td>
</tr>
<tr>
<td>$\max\left(T + r, \frac{1}{\beta} + \eta\right) &lt; \tau &lt; T + \frac{1}{\beta} + \eta$</td>
<td>$\frac{\beta}{\sqrt{1 - \beta^2}} (\tau - r)$</td>
<td>0</td>
</tr>
</tbody>
</table>

Here we use the Gaussian units, $r = \sqrt{\rho^2 + z^2}$, $z_\rho = \frac{z - \beta \rho \tau}{\sqrt{1 - \beta^2}}$, $r_\rho = \sqrt{\rho^2 + z^2}$, $\tau_\rho = \frac{\tau - \beta z}{\sqrt{1 - \beta^2}}$, $z_T = \frac{z - \beta \rho (\tau - T)}{\sqrt{1 - \beta^2}}$, $r_T = \sqrt{\rho^2 + z^2_T}$, $\tau_T = \frac{\tau - (\tau - \beta z)}{\sqrt{1 - \beta^2}}$, $z_l = z - l$, and $\eta = \sqrt{\rho^2 + z_l^2}$.

Let us investigate the magnetic induction in four equidistant observation points $A_1, A_2$ located as shown in Figure 1 at the distance $\rho_H$ from the terminal point $O$ on the plane perpendicular to $OzO$. It can be found geometrically that the induction components in the Cartesian coordinates $XYZ$ of Figure 1 may be expressed via general function $B_\phi(z, \rho, \tau, l)$ of equation (2) as follows:

$$B_{\phi_1}(A_1) = -B_{\phi_2}(-\rho_H + l_1, l_1, \tau, l_1), -\kappa B_{\phi_3}(-\rho_H, l_H, \tau - l_H/\beta, l_2), (1-\kappa)B_{\phi_4}(-\rho_H, l_H, \tau - l_H/\beta, l_H),$$

$$B_{\phi_2}(A_2) = (1-\kappa)B_{\phi_1}(-\rho_H, l_H, \tau - l_H/\beta, l_H),$$

$$B_{\phi_3}(A_1) = -\frac{l_H}{\sqrt{\rho_H^2 + l_H^2}} \left( B_{\phi_2}(l_H + 1, l_H, \tau, l_1) + \kappa B_{\phi_3}(0, \sqrt{\rho_H^2 + l_H^2}, \tau - l_H/\beta, l_2) \right),$$

$$B_{\phi_4}(A_2) = -\frac{l_H}{\sqrt{\rho_H^2 + l_H^2}} \left( B_{\phi_2}(l_H, \sqrt{\rho_H^2 + l_H^2}, \tau, l_1) + \kappa B_{\phi_3}(0, \sqrt{\rho_H^2 + l_H^2}, \tau - l_H/\beta, l_2) \right),$$

$$B_{\phi_2}(A_1) = -B_{\phi_2}(A_2), \quad B_{\phi_3}(A_1) = B_{\phi_3}(A_2), \quad B_{\phi_4}(A_1) = B_{\phi_4}(A_2) = 0,$$

$$B_{\phi_3}(A_2) = -B_{\phi_2}(A_2), \quad B_{\phi_4}(A_1) = B_{\phi_4}(A_2) = 0,$$

$$B_{\phi_4}(A_1) = -B_{\phi_2}(A_2), \quad B_{\phi_3}(A_2) = B_{\phi_3}(A_1), \quad B_{\phi_2}(A_2) = -B_{\phi_2}(A_1).$$
Time dependencies of the magnetic induction components for the case of \( l_1=5 \) km, \( l_2=15 \) km, \( l_H=5 \) km, \( T=30 \) km, \( \beta=0.8 \), three values of the branching current share \( \kappa=0.25, 0.75, \) and \( 1 \) (pure cloud-to-cloud discharge), and two distances of the observation point \( \rho_H=15 \) km and \( 50 \) km are shown in Figure 2. \( B_\phi \) is represented on the plots by the dimensionless parameter \( B_\phi = \frac{c}{\rho_H} B_\phi \). For the sake of simplicity we take the triangle current pulse
\[
I(z, \tau) = 1 - \frac{1}{\beta \tau}(\beta \tau - z)
\]
that allow analytical representation of the field [Borisov and Kononov, 1996]. However, one can use relations (2) and (3) for the current pulse of arbitrary shape.

The results obtained for the magnetic field in free space can be readily extended to another limiting case: ideally conducting earth’s surface. Here the normal component is zero while the tangential components are increased two-fold

\[
B_{\phi,x} \rightarrow 2B_{\phi,x}, \quad B_{\phi,y} \rightarrow 2B_{\phi,y}.
\]

Generalization of the discussed scheme to the case of several branches and arbitrary location of the observation point \( A \) is also possible. However, it results in much more complicated expressions. It should be noted that the horizontal part of the branching current can be represented as a sum of the pulse \( \kappa I(z, \tau) \) propagating along the path \( l_0= l_1+ l_2 \) and the pulse \((1-\kappa)I(z, \tau) \) propagating along \( l_1 \). Total induction due to the horizontal current can be obtained with the help of function

\[
\frac{\partial u}{\partial \tau} = \frac{\kappa}{c} \int (\Phi_2(z, \rho, \tau, l_0) d\zeta \right) \frac{1}{\sqrt{\rho^2 + (z-z')^2}} I(z', \tau - \sqrt{\rho^2 + (z-z')^2})
\]

\[
+ \frac{1-\kappa}{c} \int (\Phi_1(z, \rho, \tau, l_1) d\zeta \right) \frac{1}{\sqrt{\rho^2 + (z-z')^2}} I(z', \tau - \sqrt{\rho^2 + (z-z')^2})
\]

which is easier to extend to the case of several branches than the ordinary relation resulting from consideration of pulses \( \kappa I(z, \tau) \) and \( \kappa I(z, \tau - /\beta) \) propagating along the segments \( l_1 \) and \( l_2 \).

Note that calculation of the magnetic induction component allows defining the electric strength component of the discharge field in the far zone.

REFERENCES
Figure 2. Calculated lightning magnetic induction components illustrating variation of the waveforms for different observation points and values of the branching parameter $\kappa$; $l_1=5$ km, $l_2=15$ km, $l_H=5$ km, $T=30$ km, and $\beta=0.8$. 