“Days on Diffraction” is an annual conference taking place in St. Petersburg since 1968. The event is organized in May–June by St. Petersburg University, St. Petersburg Department of Steklov Mathematical Institute and Euler International Mathematical Institute of the Russian Academy of Sciences.

About 180 scientists come from different parts of the world to participate in “Days on Diffraction” 2012; the Organizing Committee thanks all of them. Of special gratitude are the authors of extended abstracts submitted to the Proceedings; 51 of them (selected by peer-review) are published in the present issue.


Web site of the conference: http://www.pdmi.ras.ru/~dd/

The conference is sponsored by

Russian Foundation for Basic Research
IEEE Russia (Northwest)
Russian Academy of Sciences
St. Petersburg State University
The 125th anniversary of V.I. Smirnov’s birthday

The 125th anniversary of Vladimir Ivanovich Smirnov’s birthday occurs on 10 June 2012. He is known not only as a prominent researcher in the fields of mathematics and mechanics, but also as a reformer of the mathematical instruction at the Leningrad (now St. Petersburg) University. His interest in mathematics dates back to the years of his education at the St. Petersburg Gymnasium no. 2, where his schoolmates were A.A. Friedmann and J.D. Tamarkin. At the St. Petersburg University, they formed the nucleus of a group of brilliant disciples of V.A. Steklov. Many years later, Smirnov accomplished the creation of the famous Leningrad school of Mathematical Physics that had been initiated by Steklov. A significant role in the formation of this school belongs to the 5-volume “A Course of Higher Mathematics”, for which Smirnov was awarded the Stalin prize for 1948 (later renamed as the State prize).

During his long career at the Leningrad University, Smirnov headed many departments. Some of them he organized himself, in particular, the renowned Departments of Mathematical Physics within the Faculties of Physics and of Mathematics and Mechanics. He headed both of them until the death in 1974, when L.D. Faddeev (famous for many discoveries in theoretical physics that include Faddeev equations of the quantum three-body problem, Faddeev–Popov ghosts etc.) and N.N. Ural’tseva (well-known for her fundamental results concerning nonlinear partial differential equations) became his successors at the Physics and Mathematics and Mechanics faculties, respectively.

Smirnov’s works are classics in various fields of mathematics. Best known are his contributions to complex analysis and the mathematical theory of diffraction. His method of functionally invariant solutions (it was developed in collaboration with S.L. Sobolev), that allowed to obtain explicit solutions for a number of important problems for the wave equation in domains with plane boundaries, is still developing further at present.
Contents

Eron L. Aero, Anatolii N. Bulygin, Yurii V. Pavlov
Exact solutions of nonlinear Klein–Fock–Gordon equation ........................................... 7

Irina L. Alexandrova, Nikolai B. Pleshchinskii
Scanning periodic grating: diffraction problem and transmission problem ......................... 13

Mikhail V. Altaisky, Natalia E. Kaputkina
On wavelet transform in Minkowski space ............................................................................. 17

A.V. Anufrieva, D.N. Tumakov, V.L. Kipot
Elastic wave propagation through a layer with graded-index distribution of density ............... 21

V.M. Babich, B.A. Samokish, N.V. Mokeeva
Diffraction of a plane wave by a transparent wedge. Numerical approach ........................... 27

Baranov D.G., Vinogradov A.P., Simovski C.R.
Perfect absorption by semi-infinite indefinite medium ............................................................ 32

Yuriy N. Belyayev
Calculations of transfer matrix by means of symmetric polynomials .................................... 36

V.V. Borzov, E.V. Damaskinsky
The differential equations for generalized parametric Chebyshev polynomials ...................... 42

Vitalii N. Chukov
The new laws of the Rayleigh wave scattering on a near-surface inhomogeneity ..................... 47

Dmitrieva L.A., Chepilko S.S.
Reduction of the Ito functional integral associated with two-dimensional non-constant diffusion process with drift to the Wiener type path integral .................................................. 54

Sergey Dobrokhotov, Michel Rouleux
The semi classical Maupertuis–Jacobi correspondence: stable and unstable spectra ............... 59

Victor G. Farafonov, Maria V. Sokolovskaya, Vladimir B. Il’in
Solution of the electrostatic problem for a non-confocal core-mantle spheroid ...................... 65

George V. Filippenko
The forced oscillations of the cylindrical shell partially submerged into a layer of liquid ........... 70

Yuriy V. Gandel’, Vladimir D. Dushkin
The method of parametric representations of integral and pseudo-differential operators in diffraction problems on electrodynamic structures ......................................................... 76

Larisa A. Glushchenko, Fedor A. Zapryagaev, Vladimir S. Makin, Vadim Ya. Krokhalev
Human body surface oscillations remote measurements by means of laser Doppler interferometer . 82

Evgeny V. Glushkov, Natalia V. Glushkova, Mikhail V. Golub, Jochen Moll, Claus-Peter Fritzen
Elastic wave energy trapping in a plate with a crack: theory and experiment .......................... 86

Mikhail V. Golub, Chuanzeng Zhang
Transmission and resonances in layered phononic crystals with damages ............................. 92

Leonid I. Goray
Energy-absorption calculus for multi-boundary conical-diffraction gratings ........................... 98
<table>
<thead>
<tr>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Evgeny A. Gorodnitskiy, Maria V. Perel, Yu Geng, Ru-Shan Wu</td>
<td>104</td>
</tr>
<tr>
<td>Poincaré wavelet techniques in depth migration</td>
<td></td>
</tr>
<tr>
<td>V.A. Gusev</td>
<td>111</td>
</tr>
<tr>
<td>Theory of selfrefraction effect of intensive focused acoustical beams</td>
<td></td>
</tr>
<tr>
<td>Sönke Hansen</td>
<td>115</td>
</tr>
<tr>
<td>The surface impedance tensor and Rayleigh waves</td>
<td></td>
</tr>
<tr>
<td>Andrey V. Ivanov, Alexander N. Shalygin, Petr E. Vorobev, Sergey S. Vergeles, Andrey K. Sarychev</td>
<td>119</td>
</tr>
<tr>
<td>Plasmon excitation in array of adjoining metal nanorods: field enhancement and optical sensing</td>
<td></td>
</tr>
<tr>
<td>Aleksei P. Kiselev, Alexandr B. Plachenov, Pedro Chamorro-Posada</td>
<td>124</td>
</tr>
<tr>
<td>Further generalizations of the Bateman solution. Novel wave beams and wave packets</td>
<td></td>
</tr>
<tr>
<td>Yaroslav Y. Konovalov</td>
<td>129</td>
</tr>
<tr>
<td>Iterative algorithms for computation convolutions of atomic functions including new family $c_{\alpha,\beta}(x)$</td>
<td></td>
</tr>
<tr>
<td>Rafał Kotyński</td>
<td>134</td>
</tr>
<tr>
<td>Metal–dielectric layered metamaterials for sub-diffraction spatial filtering of the optical wavefront</td>
<td></td>
</tr>
<tr>
<td>I.P. Krasnov</td>
<td>139</td>
</tr>
<tr>
<td>The concept of sources in electrodynamics and its application to diffraction problems</td>
<td></td>
</tr>
<tr>
<td>I.P. Krasnov</td>
<td>145</td>
</tr>
<tr>
<td>The main vectors among those used for description of electromagnetic field</td>
<td></td>
</tr>
<tr>
<td>Victor F. Kravchenko, Dmitry V. Churikov</td>
<td>152</td>
</tr>
<tr>
<td>Integrated nonparametric estimations of probability density of stochastic processes by atomic functions</td>
<td></td>
</tr>
<tr>
<td>Victor F. Kravchenko, Dmitry V. Churikov</td>
<td>158</td>
</tr>
<tr>
<td>New constructions of digital filters synthesis on base of generalized Kravchenko–Kotelnikov sampling theorem</td>
<td></td>
</tr>
<tr>
<td>Alexander V. Kudrin, Natalya M. Shmeleva, Tatyana M. Zaboronkova</td>
<td>163</td>
</tr>
<tr>
<td>Excitation of electromagnetic waves by a pulsed ring electric current in a magnetoplasma containing a cylindrical density duct</td>
<td></td>
</tr>
<tr>
<td>Andrey A. Kukushkin</td>
<td>169</td>
</tr>
<tr>
<td>On homogenization of the periodic Dirac operator</td>
<td></td>
</tr>
<tr>
<td>Alexander G. Kyurkchan, Dmitry B. Demin</td>
<td>174</td>
</tr>
<tr>
<td>Pattern equation method and T-matrix method for averaging scattering characteristics in a 2D diffraction problem</td>
<td></td>
</tr>
<tr>
<td>Rayisa P. Moiseyenko, Sarah Benchabane, Vincent Laude, Jingfei Liu, Nico F. Declercq</td>
<td>178</td>
</tr>
<tr>
<td>Scholte–Stoneley waves on 2D phononic crystal gratings</td>
<td></td>
</tr>
<tr>
<td>Kateryna V. Nesvit</td>
<td>183</td>
</tr>
<tr>
<td>Hypersingular integral equation of wave diffraction problem on pre-Cantor grating and its discrete mathematical models</td>
<td></td>
</tr>
<tr>
<td>Parfenyev V.M., Vergeles S.S.</td>
<td>188</td>
</tr>
<tr>
<td>Spaser in above-threshold regime: the lasing frequency shift</td>
<td></td>
</tr>
<tr>
<td>F. Pastrone, J. Engelbrecht</td>
<td>192</td>
</tr>
<tr>
<td>Waves and complexity of microstructured solids</td>
<td></td>
</tr>
</tbody>
</table>
Pavel S. Petrov, Mikhail Yu. Trofimov, Alyona D. Zakharenko
Mode parabolic equations for the modeling of sound propagation in 3D-varying shallow water waveguides .............................................. 197

Popov S.I., Gavrilov M.I., Popov I.Yu.
Localized two-particle states in deformed nanolayers .................................................. 203

Anatoliy M. Radin, Vyacheslav N. Kudashov, Alexandr B. Plachenov
New type of unstable optical resonators ................................................................. 207

Leonid L. Samoylov, Valeriy N. Trukhin, Anton S. Buyskikh, Denis P. Horkov
Edge diffraction in the scattering of focused terahertz radiation ................................ 211

Nikita N. Senik
On homogenization for periodic elliptic second order differential operators in a strip .... 215

Marcin Stolarek, Rafał Kotyński
Asymmetric transmission through a structure consisting of two photonic bandgap materials ... 221

I.G. Svechnikov
Anisotropic diffraction in acoustic delay lines with mosaic transducers .......................... 225

Dmitry A. Usanov, Sergey A. Nikitov, Alexander V. Skripal, Denis V. Ponomarev
Application of one-dimensional microwave photonic crystals for measurements of parameters of structures based on thin semiconductor layers ........................................... 229

Andrei B. Utkin
Ultrashort radiation pulses generated by laser wakefield accelerators: a time-domain approach . 234

Farkhat F. Valiev
Electromagnetic field formed by collimated gamma quanta pulse beam .......................... 237

Nadezhda K. Vdovicheva, Alexandr G. Sazontov
Numerical simulation of multipactor discharge on a dielectric surface ................................ 240

N. F. Yashina, T. M. Zaboronkova
The electromagnetic waves guided by the stratified composite media ............................ 245

B. Zeković
Example of n-ary bialgebra ......................................................................................... 250

Marina G. Zhuchkova, Daniil P. Kouzov
Resonance effects in propagation of flexural–gravity waves for a supported elastic plate floating on water .................................................................................. 253

Pavel E. Znak
Differential equation for geometrical spreading on a ray and second derivatives of eikonal matrix structure ................................................................................. 259

Author index .................................................................................................................. 262
Ultrashort radiation pulses generated by laser wakefield accelerators: A time-domain approach

Andrei B. Utkin
INOV - Inesc Inovação and ICEMS, IST, Technical University of Lisbon, Portugal;
e-mail: andrei.utkin@inov.pt

Time-domain investigation of formation of the laser-betatron radiation was carried out within the framework of a simple (toy) model of a two-component modulated line source current describing rectilinear propagation of the electron bunch perturbed by transverse oscillation in the co-propagating wakefield. Such an approach yields sufficiently general one-integral solution for the space-time structure of the emanated wave.

1 Introduction

A novel experimental technique, in which charged particles are pushed by the electric field of a plasma wave (the wakefield) driven by an intense laser, offer compact accelerators nearly as efficient as huge traditional synchrotrons [1]. The resulting extremely bright betatron radiation sources have a potential to boost numerous uses across the whole spectrum of light-source applications [2]. Due to sufficiently complicated nature of the involved processes, most of the theoretical description of such laser-driven betatrons has been done on the basis of numerical simulation (see, e.g., [3] and references therein). A few published analytical results concerning the electron/ion motion [4] are based on simplified one-dimensional models of laser piston maintained by the radiation pressure. In all circumstances (even if the early stage involves a space-time technique, such as introduction of the Linard-Wiechert potential), the betatron radiation is described using the frequency-domain approach [5].

This preliminary investigation introduces a general approach to the time-domain description of formation of the laser-betatron radiation. This radiation is launched by a source-current pulse in the form of an electron bunch, whose nearly rectilinear propagation in the direction of the primary laser pulse (taken as the z direction) is disturbed by transversal oscillation in the co-propagating wakefield.

2 Statement of the electromagnetic problem

Let us suppose that linearly polarized laser radiation generates an electron bunch propagating in the z direction at the velocity $\beta$ and oscillating in the x direction at the phase velocity $u$ (both $\beta$ and $u$ are dimensionless, normalized by the speed of light $c$), with the spatial period $2\pi/k$, as illustrated in Fig. 1. Such a geometry is well discussed in the literature since 1990s: see, for example, Figs. 1, 2 of [6] and a more contemporary model illustrated in Fig. 3a of [1]. In this case the current density $j = j_x e_x + j_z e_z$ with the amplitudes $j_{0x}$ and $j_{0z}$ can be described by

\[
\begin{pmatrix}
  j_x \\
  j_z
\end{pmatrix} = \begin{pmatrix}
  j_{0x} e^{i k (u \tau - z)} \\
  j_{0z}
\end{pmatrix} I_{4D} (\tau, x, y, z)
\times h(z) h \left( \tau - \frac{z}{\beta} \right) h \left( \frac{z}{\beta} - \tau + T \right).
\]

(1)

Here $x$, $y$, and $z$ are the Cartesian coordinates, $\tau = ct$ and $T$ the time variable and the current

Figure 1: Simplified representation of the source-current pulse generated by a superstrong linearly polarized driving laser pulse: An electron bunch whose nearly rectilinear propagation in the direction of the laser pulse is perturbed by the transverse oscillation in the co-propagating wakefield.
pulse duration represented in units of length, and $I_{4D}(\tau, x, y, z)$ is the spatiotemporal current density distribution within its support defined by the unit step functions $h(\cdot)$.

Depending on the experimental conditions, the characteristic path traveled by the current pulse before its decay varies from centimeters [5] to meters [1] while the amplitude of the transversal bunch oscillation usually does not exceed several micrometers (see, e.g., Fig. 2 of [5]). As so, for an observer located in the far zone the space-time structure $I_{4D}$ can be roughly described by a line-current model, i.e., assuming the transversal distribution $\delta(x)\delta(y)$, where $\delta(\cdot)$ is the Dirac delta function. From here on we will use the dimensionless space-time described by $\tau$ and the Cartesian $(x, y, z)$ or cylindrical $(\rho, \varphi, z)$ coordinates—all except $\varphi$ are normalized by the characteristic length $k^{-1}$. In view of the line-current model, expression (1) yields a simplified description of the current

$$
\left( \frac{j_x}{j_y} \right) = \left( \frac{j_{oz} \exp(i(\omega \tau - z))}{j_{oz}} \right) e \kappa k^3 I(\tau, z)
$$

$$
\times \delta(\rho) h(z) \left( \frac{\tau - z}{\beta} \right) h \left( \frac{z}{\beta} - \tau + T \right),
$$

where $e$ is the elementary charge and the dimensionless function $I(\tau, z)$ represents the current distribution along the propagation path.

Noticing that the source current appears at $\tau = 0$ and no electromagnetic field is expected to be generated at earlier moments of time, one can find the vector potential $A$ corresponding to the emitted wave as a solution to the initial value problem

$$
\Box A = \mu_0 j, \quad A = 0 \quad \text{for} \quad \tau < 0, \quad (2)
$$

where $\Box$ is the d'Alembertian operator and $\mu_0$ the magnetic constant. The problem for two non-zero Cartesian components $A_x$ and $A_z$ (normalized by $\mu_0 c \kappa k$) can be written as

$$
\begin{align*}
\left[ \partial^2_{\tau} - \frac{\partial^2_{\rho}}{\rho} - \frac{1}{\rho} \partial_{\rho} \left( \rho \partial_{\rho} \right) \right] A_{x,z}(\tau, \rho, z) &= \delta(\rho) J_{x,z}(\tau, z) \\
&\times h(z) \left( \frac{\tau - z}{\beta} \right) h \left( \frac{z}{\beta} - \tau + T \right), \\
A_{x,z}(\tau, \rho, z) &= 0 \quad \text{for} \quad \tau < 0,
\end{align*}
$$

where

$$
J_x(\tau, z) = j_{ox} \exp(i(\omega \tau - z)) I(\tau, z),
$$

$$
J_z(\tau, z) = j_{oz} I(\tau, z).
$$

### 3 Solution

Problem (3) can be solved using the incomplete separation of variables and the Riemann-Volterra formula as described in Sec. 4.3 of Ref. [7] yielding $A_{x,z} = 0$ for $-\infty < \tau < r$ and the one-integral formula

$$
A_{x,z} = \frac{1}{4\pi} \int_{\zeta}^{\infty} \frac{J_{x,z}(\tau - r', z')}{r'} dz'
$$

for $r < \tau < \infty$, where

$$
\zeta = \max(0, \zeta_T) = \begin{cases} 0, & r < \tau < r + T \\ \zeta_T, & r + T < \tau < \infty \end{cases},
$$

$$
\zeta_T = \frac{\beta}{\tau - T - \beta z + \sqrt{(1 - \beta^2) \rho^2 + (z - \beta (\tau - T))^2}}
$$

$$
\zeta_0 = \zeta_T|_{T=0} = \frac{\rho^2 - z'^2}{\tau - \beta z + \sqrt{(1 - \beta^2) \rho^2 + (z - \beta \tau)^2}}
$$

$$
r = \sqrt{\rho^2 + z^2}, \quad r' = \sqrt{\rho^2 + (z - \tau')^2}.
$$

In the Cartesian coordinates, due to $B = \nabla \times A$, the components of the magnetic field (normalized by $\mu_0 c \kappa k^2$) are defined by the relationships

$$
B_x(\tau, x, y, z) = \partial_y A_z, \quad B_y(\tau, x, y, z) = 0, \quad B_z(\tau, x, y, z) = -\partial_x A_y.
$$

Bearing in mind that $\partial_{\varphi} A_{x,y} \equiv 0$ and thus for the case in question

$$
\partial_x = \frac{\partial}{\partial x} = \frac{x}{\rho} \partial_{\rho} = \cos \varphi \partial_{\rho},
$$

$$
\partial_y = \frac{\partial}{\partial y} = \frac{y}{\rho} \partial_{\rho} = \sin \varphi \partial_{\rho},
$$

the Cartesian components of the magnetic field can be conveniently expressed as functions of the cylindrical coordinates

$$
B_x(\tau, \rho, \varphi, z) = \sin \varphi \partial_{\rho} A_z(\tau, \rho, z),
$$

$$
B_y(\tau, \rho, \varphi, z) = 0,
$$

$$
B_z(\tau, \rho, \varphi, z) = -\cos \varphi \partial_{\rho} A_z(\tau, \rho, z),
$$

$$
B_x(\tau, \rho, \varphi, z) = -\sin \varphi \partial_{\rho} A_x(\tau, \rho, z).
$$
4 Discussion

General solutions (4) and (5) depict the magnetic field only. In the far zone the complete electromagnetic field and the energy flux can be found using the approximations described in Ch. 9 of [8].

The electric field can be found on the basis of a more advanced model that includes—apart from the current distribution and the magnetic potential—the space-time structure of the generated electric charge \( \varrho \) and the electric potential \( \phi \). In this case \( j \) and \( \varrho \) must be linked via the continuity equation while \( A \) and \( \phi \) via the Lorenz gauge. The sketch of such an approach, applied to description of a localized wave generation by superluminal electric charges, can be found in [9].

The results obtained can be easily extended to the case of the circular or elliptic polarization of the driving laser pulse, resulting in a slightly more complicated model involving all tree components of the source current density and the vector potential.

Despite the general nature of the solution, expressions (4) and (5) enable some analysis of the electromagnetic pulse structure to be made. In particular, the form of integral (4) points out that the emanated wave must have, at least for \( \beta \rightarrow 1 \), strong directionality along the propagation direction, see [7] and a simpler analysis made in [10] for the particular case of the exponentially decaying source current.

Substituting (4) into (5) and harnessing Leibniz’s rule for differentiation under the integral sign, one can easily obtain more explicit representations to the potential derivatives \( \partial_x A_x \), \( \partial_z A_z \) and \( \partial_y A_y \). Remarkably, for the expression defining \( B_y \) (and containing the term \( \partial_z A_z \)) this procedure yields an additional term due to the extra factor \( \exp \left( i k (u \tau - z) \right) \), which may provide the most significant input to the total field amplitude and energy flux, making the effect of the transversal oscillation of the electron bunch unneglectable.

Acknowledgements

The research was partially based on work supported by the International Science Foundation under Grant M3H000 Generation and Propagation of Localized Waves.

References


